1 微分/整関数

次の関数を微分せよ．

(1) $f(x) = 2x^8 + 4x^2 + 5x - 8$ $\boldsymbol{f'(x) = 16x^7 + 8x + 5}$

(2) $f(x) = -x^3 + x^2 - 2x - 3$ $\boldsymbol{f'(x) = -3x^2 + 2x - 2}$

(3) $f(x) = -5x^2 + 5x + 6$ $\boldsymbol{f'(x) = -10x + 5}$

(4) $f(x) = -2x^8 - x^4 + 4$ $\boldsymbol{f'(x) = -16x^7 - 4x^3}$

(5) $f(x) = -x^3 - 3x^2 - 4x - 7$ $\boldsymbol{f'(x) = -3x^2 - 6x - 4}$

(6) $f(x) = 3x^2 + 4x + 5$ $\boldsymbol{f'(x) = 6x + 4}$

(7) $f(x) = -x^{10} + x^6 + 3x^2 - 7$ $\boldsymbol{f'(x) = -10x^9 + 6x^5 + 6x}$

(8) $f(x) = -x^3 + 4x^2 - 5$ $\boldsymbol{f'(x) = -3x^2 + 8x}$

(9) $f(x) = -x^3 - x^2 + 7$ $\boldsymbol{f'(x) = -3x^2 - 2x}$

(10) $f(x) = 4x^2 - 2x$ $\boldsymbol{f'(x) = 8x - 2}$

2 微分/整関数の微分

次の関数を微分せよ．

(1) $f(x) = \dfrac{1}{x^4}$

$f(x) = x^{-4}$ より $f'(x) = (-4)x^{-5}$ $\boldsymbol{f'(x) = -\dfrac{4}{x^5}}$

(2) $f(x) = -\dfrac{1}{x^3} + \dfrac{3}{x^2}$

$f(x) = -x^{-3} + 3x^{-2}$ より $f'(x) = (-4)x^{-5}$ $\boldsymbol{f'(x) = \dfrac{3}{x^4} - \dfrac{6}{x^3}}$

(3) $f(x) = \sqrt[4]{x}$

$f(x) = x^{\frac{1}{4}}$ より $f'(x) = \dfrac{1}{4}x^{-\frac{3}{4}}$ $\boldsymbol{f'(x) = \dfrac{1}{4\sqrt[4]{x^3}}}$

(4) $f(x) = \dfrac{1}{(2x+3)^2}$

$f(x) = (2x+3)^{-2}$ より $f'(x) = (-2)(2x+3)^{-3}(2x+3)'$ $\boldsymbol{f'(x) = -\dfrac{4}{(2x+3)^3}}$

(5) $f(x) = \sqrt[10]{x}$

$f(x) = x^{\frac{1}{10}}$ より $f'(x) = \dfrac{1}{10}x^{-\frac{9}{10}}$ $\boldsymbol{f'(x) = \dfrac{1}{10\sqrt[10]{x^9}}}$

(6) $f(x)=\dfrac{1}{x^2-1}$

$f(x)=(x^2-1)^{-1}$ より $f'(x)=(-1)(x^2-1)^{-2}(x^2-1)'$　　**$f'(x)=-\dfrac{2x}{(x^2-1)^2}$**

(7) $f(x)=\dfrac{1}{\sqrt{x}}$

$f(x)=(x)^{-\frac{1}{2}}$ より $f'(x)=\left(-\dfrac{1}{2}\right)x^{-\frac{3}{2}}$　　**$f'(x)=-\dfrac{1}{2x\sqrt{x}}$**

3　微分/積の微分

次の関数を微分せよ.

(1) $f(x)=(x-2)(4x+7)$

$f'(x)=1\cdot(4x+7)+(x-2)\cdot 4$　　**$f'(x)=8x-1$**

(2) $f(x)=(x-5)(x+6)$　　**$f'(x)=2x+1$**

(3) $f(x)=(2x-3)(x+4)$　　**$f'(x)=4x+5$**

(4) $f(x)=x(2x-3)$　　**$f'(x)=4x-3$**

(5) $f(x)=(2x+7)\left(2x^2+x+2\right)$　　**$f'(x)=12x^2+32x+11$**

(6) $f(x)=(x-3)\left(x^2+3x+1\right)$　　**$f'(x)=3x^2-8$**

(7) $f(x)=(x+5)\left(2x^2+x+4\right)$　　**$f'(x)=6x^2+22x+9$**

(8) $f(x)=(4x-9)\left(x^2+3x+7\right)$　　**$f'(x)=12x^2+6x+1$**

(9) $f(x)=(2x+5)\left(2x^2+x-6\right)$　　**$f'(x)=12x^2+24x-7$**

(10) $f(x)=(x-2)\left(2x^2+3x-4\right)$　　**$f'(x)=6x^2-2x-10$**

4　微分/商の微分 1

次の関数を微分せよ.

＜（解答コメント）途中式は解答例. 分母を (-1) 乗と見ることで, 積の微分で解いても良い. ＞

(1) $f(x)=\dfrac{3}{x+2}$

$f'(x)=\dfrac{0\cdot(x+2)-3\cdot 1}{(x+2)^2}$

別解 $f(x)=3(x+2)^{-1}$ より $f'(x)=3(-1)(x+2)^{-2}$　　**$f'(x)=\dfrac{-3}{(x+2)^2}$**

(2) $f(x)=\dfrac{x-2}{x+2}$ 　　$\boldsymbol{f'(x)=\dfrac{4}{(x+2)^2}}$

(3) $f(x)=\dfrac{2}{x+1}$ 　　$\boldsymbol{f'(x)=\dfrac{-2}{(x+1)^2}}$

(4) $f(x)=\dfrac{2x-1}{x+3}$ 　　$\boldsymbol{f'(x)=\dfrac{7}{(x+3)^2}}$

(5) $f(x)=\dfrac{x+3}{x-1}$ 　　$\boldsymbol{f'(x)=\dfrac{-4}{(x-1)^2}}$

(6) $f(x)=\dfrac{2x+3}{3x+5}$ 　　$\boldsymbol{f'(x)=\dfrac{1}{(3x+5)^2}}$

(7) $f(x)=\dfrac{3x+1}{x^2+5}$ 　　$\boldsymbol{f'(x)=\dfrac{-3x^2-2x+15}{(x^2+5)^2}}$

(8) $f(x)=\dfrac{x-3}{x^2-x}$ 　　$\boldsymbol{f'(x)=\dfrac{-x^2+6x-3}{(x^2-x)^2}}$

(9) $f(x)=\dfrac{3x}{x^2+2x-3}$ 　　$\boldsymbol{f'(x)=\dfrac{-3x^2-9}{(x^2+2x-3)^2}}$

(10) $f(x)=\dfrac{x+3}{\sqrt{x}}$

$$f'(x)=\frac{1\cdot\sqrt{x}-(x+3)(\sqrt{x})^2}{\left(\sqrt{x}\right)^2}=\frac{\sqrt{x}-(x+3)\frac{1}{2\sqrt{x}}}{x}=\frac{2x-(x+3)}{2x\sqrt{x}}$$

$$\boldsymbol{f'(x)=\frac{x-3}{2x\sqrt{x}}}$$

(11) $f(x)=\dfrac{3x^2-2x+3}{\sqrt{x}}$

$$\begin{aligned}f'(x)&=\frac{(6x-2)\sqrt{x}-(3x^2-2x+3)\left(\sqrt{x}\right)'}{\left(\sqrt{x}\right)^2}\\&=\frac{(6x-2)\sqrt{x}-(3x^2-2x+3)\cdot\frac{1}{2\sqrt{x}}}{x}\\&=\frac{(6x-2)\cdot 2x-(3x^2-2x+3)}{2x\sqrt{x}}\end{aligned}$$

$$\boldsymbol{f'(x)=\frac{9x^2-2x-3}{2x\sqrt{x}}}$$

5　微分/商の微分 2

次の関数を微分せよ.

＜（解答コメント）商の微分を回避できる問題. 積の微分で処理したほうが計算は速いかも？＞

(1) $f(x)=\dfrac{1}{2x}$

$f(x)=\dfrac{1}{2}x^{-1}$ より

$f'(x)=\dfrac{1}{2}\cdot(-1)x^{-2}$ $\qquad$ $\boldsymbol{f'(x)=-\dfrac{1}{2x^2}}$

(2) $f(x)=\dfrac{1}{3x-1}$

$f(x)=(3x-1)^{-1}$ より

$f'(x)=(-1)(3x-1)^{-2}\cdot 3$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-3}{(3x-1)^2}}$

(3) $f(x)=\dfrac{1}{x^2+3x+3}$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-2x-3}{(x^2+3x+3)^2}}$

(4) $f(x)=\dfrac{1}{x^2-x+5}$

$f(x)=(x^2-x+5)^{-1}$ より

$f'(x)=(-1)(x^2-x+5)^{-2}\cdot\underbrace{(x^2-x+5)'}_{2x-1}$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-2x+1}{(x^2-x+5)^2}}$

(5) $f(x)=\dfrac{1}{x^2+3}$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-2x}{(x^2+3)^2}}$

(6) $f(x)=\dfrac{1}{x^2-x+3}$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-2x+1}{(x^2-x+3)^2}}$

(7) $f(x)=\dfrac{1}{x^2-x-2}$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-2x+1}{(x^2-x-2)^2}}$

(8) $f(x)=\dfrac{1}{x^2+3x-4}$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-2x-3}{(x^2+3x-4)^2}}$

(9) $f(x)=\dfrac{1}{x^2-2x+1}$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-2}{(x-1)^3}}$

(10) $f(x)=\dfrac{1}{x^2-3x}$ $\qquad$ $\boldsymbol{f'(x)=\dfrac{-2x+3}{(x^2-3x)^2}}$

6　微分/合成関数の微分

次の関数を微分せよ.

(1) $f(x)=(x^2+2)^7$

$f'(x)=7(x^2+2)^6\cdot\underbrace{(x^2+2)'}_{2x}$ $\qquad$ $\boldsymbol{f'(x)=14x(x^2+2)^6}$

(2) $f(x) = (2x^2 - 1)^4$ $\qquad$ $\boldsymbol{f'(x) = 16x(2x^2 - 1)^3}$

(3) $f(x) = (x + 1)^3$ $\qquad$ $\boldsymbol{f'(x) = 3(x + 1)^2}$

(4) $f(x) = (x^2 + x + 1)^6$ $\qquad$ $\boldsymbol{f'(x) = 6(x^2 + x + 1)^5(2x + 1)}$

(5) $f(x) = (x - 2)^3$ $\qquad$ $\boldsymbol{f'(x) = 3(x - 2)^2}$

(6) $f(x) = (x^2 + 3x - 1)^8$ $\qquad$ $\boldsymbol{f'(x) = 8(x^2 + 3x - 1)^7(2x + 3)}$

(7) $f(x) = (x^2 - 1)^5$ $\qquad$ $\boldsymbol{f'(x) = 10x(x^2 - 1)^4}$

(8) $f(x) = \dfrac{1}{(x^2 + 1)^5}$

$f(x) = (x^2 + 1)^{-5}$ より

$$f'(x) = (-5)(x^2 + 1)^{-6} \cdot \underbrace{(x^2 + 1)'}_{2x} \qquad \boldsymbol{f'(x) = -\frac{10x}{(x^2 + 1)^6}}$$

(9) $f(x) = \dfrac{1}{\sqrt{x^2 + 1}}$

$f(x) = \left(x^2 + 1\right)^{-\frac{1}{2}}$ より

$$f'(x) = -\tfrac{1}{2}\left(x^2 + 1\right)^{-\frac{3}{2}} \cdot \underbrace{(x^2 + 1)'}_{2x} \qquad \boldsymbol{f'(x) = -\frac{x}{(x^2 + 1)\sqrt{x^2 + 1}}}$$

(10) $f(x) = \left(x + \dfrac{1}{x}\right)^3$

$$f'(x) = 3\left(x + \frac{1}{x}\right)^2 \cdot \underbrace{\left(x + x^{-1}\right)'}_{1+(-1)x^{-2}} \qquad \boldsymbol{f'(x) = 3\left(x + \frac{1}{x}\right)^2\left(1 - \frac{1}{x^2}\right)}$$

(11) $f(x) = (x - 3)^3(x + 5)^4$

$f'(x) = 3(x - 3)^2(x + 5)^4 + 4(x + 5)^3(x - 3)^3$

$$\boldsymbol{f'(x) = (7x + 3)(x - 3)^2(x + 5)^3}$$

(12) $f(x) = (2x^2 + 5)^3(x^2 - 5)^7$

$f'(x) = 12x(2x^2 + 5)^2(x^2 - 5)^7 + 14x(x^2 - 5)^6(2x^2 + 5)^3$

$$\boldsymbol{f'(x) = 10x(4x^2 + 1)(2x^2 + 5)^2(x^2 - 5)^6}$$

7 微分/三角関数

次の関数を微分せよ．

(1) $f(x) = \sin(2x)$

$$f'(x) = \cos 2x \cdot \underbrace{(2x)'}_{2} \qquad \boldsymbol{f'(x) = 2\cos(2x)}$$

(2) $f(x)=\cos(x-1)$

$$f'(x)=-\sin(x-1)\cdot\underbrace{(x-1)'}_{1}$$

$$\boldsymbol{f'(x)=-\sin(x-1)}$$

(3) $f(x)=\dfrac{1}{\sin x}$

$$f'(x)=\frac{0\cdot\sin x-1\cdot\cos x}{(\sin x)^2}$$

$$\boldsymbol{f'(x)=-\frac{\cos x}{\sin^2 x}}$$

(4) $f(x)=\tan\left(\sqrt{x}\right)$

$$f'(x)=\frac{1}{\cos^2\sqrt{x}}\cdot\underbrace{\left(\sqrt{x}\right)'}_{\frac{1}{2}x^{-\frac{1}{2}}}$$

$$\boldsymbol{f'(x)=\frac{1}{2\sqrt{x}\cos^2\sqrt{x}}}$$

(5) $f(x)=\dfrac{1}{\tan x}$

$$f'(x)=\frac{0\cdot\tan x-1\cdot\frac{1}{\cos^2 x}}{\tan^2 x}=\frac{-\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}}$$

$$\boldsymbol{f'(x)=-\frac{1}{\sin^2 x}}$$

(6) $f(x)=\cos\left(x^2-x+2\right)$

$$f'(x)=-\sin(x^2-x+2)\underbrace{(x^2-x+2)'}_{2x-1}$$

$$\boldsymbol{f'(x)=(-2x+1)\sin\left(x^2-x+2\right)}$$

(7) $f(x)=\sin\left(x^2-2x-2\right)$

$$\boldsymbol{f'(x)=(2x-2)\cos\left(x^2-2x-2\right)}$$

(8) $f(x)=\cos\left(2x^2-1\right)$

$$\boldsymbol{f'(x)=-4x\sin\left(2x^2-1\right)}$$

(9) $f(x)=\sin^2(x+4)$

$$f'(x)=2\sin(x+4)\cdot\underbrace{\{\sin(x+4)\}'}_{\cos(x+4)\cdot\underbrace{(x+4)'}}$$

以後, 解答が 2 つ以上書いてあることがある. どちらでも正解.(この問題では,2 倍角を使い変形した.)

$$\boldsymbol{f'(x)=2\sin(x+4)\cos(x+4)=\sin(2x+8)}$$

(10) $f(x)=\cos^3\left(x^2+1\right)$

$$f'(x)=3\cos^2(x^2+1)\cdot\underbrace{\left\{\cos(x^2+1)\right\}'}_{-\sin(x^2+1)\cdot\underbrace{(x^2+1)'}_{2x}}$$

$$\boldsymbol{f'(x)=-6x\cos^2\left(x^2+1\right)\sin\left(x^2+1\right)=-3x\cos(x^2+1)\sin(2x^2+2)}$$

(11) $f(x)=\sin^2\left(2x^2+x-1\right)$

$$f'(x)=2\sin(2x^2+x-1)\cdot\underbrace{\left\{\sin(2x^2+x-1)\right\}'}_{\cos(2x^2+x-1)\cdot\underbrace{(2x^2+x-1)'}_{4x+1}}$$

$$\boldsymbol{f'(x) = (8x+2)\sin\left(2x^2+x-1\right)\cos\left(2x^2+x-1\right) = (4x+1)\sin(4x^2+2x-2)}$$

8 微分/指数関数

次の関数を微分せよ.

(1) $f(x) = e^{x-2}$ $\boldsymbol{f'(x) = e^{x-2}}$

(2) $f(x) = e^{x^2+3}$

$f'(x) = e^{x^2+3}\cdot\underbrace{(x^2+3)'}_{2x}$ $\boldsymbol{f'(x) = 2xe^{x^2+3}}$

(3) $f(x) = e^{x^2+3x+5}$ $\boldsymbol{f'(x) = (2x+3)\,e^{x^2+3x+5}}$

(4) $f(x) = 3^x$ $\boldsymbol{f'(x) = 3^x\log 3}$

(5) $f(x) = 5^{3x+4}$

$f'(x) = 5^{3x+4}\log 5\cdot\underbrace{(3x+4)'}_{3}$ $\boldsymbol{f'(x) = 3\cdot 5^{3x+4}\log 5}$

(6) $f(x) = 3^{x^3-1}$

$f'(x) = 3^{x^3-1}\cdot\log 3\cdot\underbrace{(x^3-1)'}_{3x^2} = \left(3^{x^3-1}\cdot 3\right)\cdot x^2\cdot\log 3$ $\boldsymbol{f'(x) = x^2\cdot 3^{x^3}\log 3}$

(7) $f(x) = xe^x$

積の微分を使う.

$f'(x) = 1\cdot e^x + x\cdot e^x$ $\boldsymbol{f'(x) = e^x(1+x)}$

9 微分/対数関数

次の関数を微分せよ.

(1) $f(x) = \log\left(x^2+2\right)$

$x^2+2>0$ なので, 絶対値はついていないが微分の公式が使える.

$f'(x) = \dfrac{(x^2+2)'}{x^2+2} = \dfrac{2x}{x^2+2}$ $\boldsymbol{f'(x) = \dfrac{2x}{x^2+2}}$

(2) $f(x) = \log\left|x^2-2\right|$ $\boldsymbol{f'(x) = \dfrac{2x}{x^2-2}}$

(3) $f(x) = \log\left(2x^2+3x+4\right)$ $\boldsymbol{f'(x) = \dfrac{4x+3}{2x^2+3x+4}}$

(4) $f(x) = \log\left|x^2+x-2\right|$ $\boldsymbol{f'(x) = \dfrac{2x+1}{x^2+x-2}}$

(5) $f(x)=\log_5|2x-1|$

底の変換公式より $f(x)=\dfrac{\log|2x-1|}{\log 5}$

分母は定数であることに気をつけて，$f'(x)=\dfrac{1}{\log 5}\cdot\dfrac{(2x-1)'}{2x-1}$　　$\boldsymbol{f'(x)=\dfrac{2}{(2x-1)\log 5}}$

(6) $f(x)=\log_2|3x-1|$　　$\boldsymbol{f'(x)=\dfrac{3}{(3x-1)\log 2}}$

(7) $f(x)=\log_a\left|2x^2-2x-1\right|\quad(a>0, a\neq 1)$

底の変換公式より $f(x)=\dfrac{\log|2x^2-2x-1|}{\log a}$

$f'(x)=\dfrac{1}{\log a}\cdot\dfrac{(2x^2-2x-1)'}{2x^2-2x-1}$　　$\boldsymbol{f'(x)=\dfrac{4x-2}{(2x^2-2x-1)\log a}}$

(8) $f(x)=x^2\log x\quad(x>0)$

積の微分を使う．

$f'(x)=2x\log x+x^2\cdot\dfrac{1}{x}$　　$\boldsymbol{f'(x)=2x\log x+x=x(2\log x+1)}$

10　微分/いろいろ

次の関数を微分せよ．

(1) $f(x)=\sin^2(2x)$

$$f'(x)=2\sin 2x\cdot\underbrace{(\sin 2x)'}_{\cos 2x\cdot\underbrace{(2x)'}_{2}}$$

$\boldsymbol{f'(x)=4\sin 2x\cos 2x=2\sin 4x}$

(2) $f(x)=e^{x\log x}$

合成関数の微分．

$$f'(x)=e^{x\log x}\cdot\underbrace{(x\log x)'}_{\text{積の微分}}$$

$$=\underbrace{\left(e^{\log x}\right)}_{\log\text{の定義より},\ x}{}^{x}\cdot\left(\log x+x\cdot\frac{1}{x}\right)$$

$$=x^x(\log x+1)$$

$\boldsymbol{f'(x)=x^x(\log x+1)}$

(3) $f(x)=\cos(\sin x)$

$f'(x) = -\sin(\sin x) \cdot \underbrace{(\sin x)'}_{\cos x}$ $\boldsymbol{f'(x) = -\sin(\sin x) \cdot \cos x}$

(4) $f(x) = \sqrt{1+\sin^2 x}$

$f(x) = \left(1+\sin^2 x\right)^{\frac{1}{2}}$ より

$$\begin{aligned} f'(x) &= \frac{1}{2}\left(1+\sin^2 x\right)^{-\frac{1}{2}} \cdot \left(1+\sin^2 x\right)' \\ &= \frac{1}{2} \cdot \frac{1}{\sqrt{1+\sin^2 x}} \cdot \left\{ 2\sin x \cdot \underbrace{(\sin x)'}_{\cos x} \right\} \\ &= \frac{1}{\sqrt{1+\sin^2 x}} \cdot \sin x \cos x \end{aligned}$$

$$\boldsymbol{f'(x) = \frac{\sin x \cos x}{\sqrt{1+\sin^2 x}} = \frac{\sin 2x}{2\sqrt{1+\sin^2 x}}}$$

(5) $f(x) = \cos^3\left(2x^2+x+2\right)$

$f'(x) = 3\cos^2(2x^2+x+2) \cdot \underbrace{\left\{\cos(2x^2+x+2)\right\}'}_{-\sin(2x^2+x+1)\cdot\underbrace{(2x^2+x+1)'}_{4x+1}}$

$$\boldsymbol{f'(x) = -3\left(4x+1\right)\cos^2\left(2x^2+x+2\right)\sin\left(2x^2+x+2\right)}$$

もしくは 2 倍角を用いて $\boldsymbol{f'(x) = -\frac{3}{2}\left(4x+1\right)\cos\left(2x^2+x+2\right)\sin\left\{2\left(2x^2+x+2\right)\right\}}$

(6) $f(x) = e^{-2x}\sin 2x$

積の微分から．

$$\begin{aligned} f'(x) &= e^{-2x}\underbrace{(-2x)'}_{-2} \cdot \sin 2x + e^{-2x}\cos 2x \cdot \underbrace{(2x)'}_{2} \\ &= e^{-2x}(-2\sin 2x) + e^{-2x}(2\cos 2x \end{aligned}$$

$$\boldsymbol{f'(x) = 2e^{-2x}(\cos 2x - \sin 2x)}$$

(7) $f(x) = \sqrt[3]{x^2+1}$

$f'(x) = \frac{1}{3}(x^2+1)^{-\frac{2}{3}} \cdot \underbrace{(x^2+1)'}_{2x}$ $\boldsymbol{f'(x) = \frac{2x}{3\sqrt[3]{(x^2+1)^2}}}$

(8) $f(x) = \log\left|\frac{2x-1}{2x+1}\right|$

$f(x) = \log|2x-1| - \log|2x+1|$ だから

$$f'(x)=\frac{(2x-1)'}{2x-1}-\frac{(2x+1)'}{2x+1}$$

$$\boldsymbol{f'(x)=\frac{4}{(2x+1)(2x-1)}}$$

(9) $f(x)=\sin^5 x\cos 5x$

積の微分を使う．

$$f'(x)=\left\{5\sin^4 x\underbrace{(\sin x)'}_{\cos x}\right\}\cos 5x+\sin^5 x\left\{-\sin 5x\cdot\underbrace{(5x)'}_{5}\right\}$$

$$=5\sin^4 x\underbrace{(\cos x\cos 5x-\sin x\sin 5x)}_{\text{加法定理より}\cos(x+5x)}$$

$$\boldsymbol{f'(x)=5\sin^4 x\cos 6x}$$

(10) $f(x)=\log\dfrac{|x|}{1+\cos x}$

$f(x)=\log|x|-\log(1+\cos x)$ より

$$f'(x)=\frac{1}{x}-\frac{(1+\cos x)'}{1+\cos x}$$

$$\boldsymbol{f'(x)=\frac{1}{x}+\frac{\sin x}{1+\cos x}}$$

(11) $f(x)=\dfrac{1}{\tan^3(2x)}$

$f(x)=\tan^{-3}(2x)$ より

$$f'(x)=-3\tan^{-4}(2x)\cdot\underbrace{(\tan 2x)'}_{\frac{1}{\cos^2 2x}\cdot\underbrace{(2x)'}_{2}}$$

$$=-6\cdot\frac{1}{\tan^4 2x}\cdot\frac{1}{\cdot\cos^2 2x}$$

$$=-6\cdot\frac{\cos^4 2x}{\sin^4 2x}\cdot\frac{1}{\cos^2 2x}$$

$$=-6\cdot\frac{\cos^2 2x}{\sin^4 2x}$$

$$\boldsymbol{f'(x)=-\frac{6}{\tan^4 2x\cos^2 2x}=-\frac{6\cos^2 2x}{\sin^4 2x}=-\frac{6}{\tan^2 2x\sin^2 2x}}$$

(12) $f(x)=\dfrac{1-\tan x}{1+\tan x}$

$$f'(x)=\frac{-\frac{1}{\cos^2 x}\cdot(1+\tan x)-(1-\tan x)\cdot\dfrac{1}{\cos^2 x}}{(1+\tan x)^2}=\frac{-(1+\tan x)-(1-\tan x)}{\cos^2(1+\tan x)^2}$$

$$\boldsymbol{f'(x)=-\frac{2}{\cos^2 x(1+\tan x)^2}}$$

11 微分/対数微分法

次の関数を微分せよ．

(1) $f(x) = x^{\log x} \quad (x > 0)$

両辺の対数をとって微分する．

$$\log|f(x)| = \log x \cdot \log x = (\log x)^2$$

$$\frac{f'(x)}{f(x)} = 2\log x \underbrace{(\log x)'}_{\frac{1}{x}=x^{-1}}$$

$$\therefore f'(x) = f(x)\left(2\log x \cdot x^{-1}\right) = 2x^{\log x} \cdot x^{-1}\log x$$

$$\boldsymbol{f'(x) = 2x^{(\log x)-1}\log x}$$

(2) $f(x) = (\log x)^x \quad (x > 1)$

両辺の対数をとって微分する．($x > 1$ は両辺の対数を取るために必要な条件．とりあえず気にしなくても良い．)

$$\log|f(x)| = x \cdot \log(\log x)$$

$$\frac{f'(x)}{f(x)} = 1 \cdot \log(\log x) + x \cdot \frac{1}{\log x}\underbrace{(\log x)'}_{\frac{1}{x}} = \log(\log x) + \frac{1}{\log x}$$

$$f'(x) = f(x)\left\{\log(\log x) + \frac{1}{\log x}\right\}$$

$$\boldsymbol{f'(x) = (\log x)^x\left\{\log(\log x) + \frac{1}{\log x}\right\}}$$

(3) $f(x) = (2x-1)^4(3-x)^3$

両辺の対数をとって微分する．

$$\log|f(x)| = 4\log|2x-1| + 3\log|3-x|$$

$$\frac{f'(x)}{f(x)} = 4 \cdot \frac{(2x-1)'}{2x-1} + 3 \cdot \frac{(3-x)'}{3-x}$$

$$f'(x) = f(x) \cdot \left(\frac{8}{2x-1} + \frac{-3}{3-x}\right) = (2x-1)^4(3-x)^3\left(\frac{8}{2x-1} + \frac{-3}{3-x}\right)$$

別解 積の微分を使う．

$$f'(x) = 4(2x-1)^3(2x-1)'(3-x)^3 + (2x-1)^4 \cdot 3(3-x)^2(3-x)'$$
$$= (2x-1)^3(3-x)^2\{8(3-x) - 3(2x-1)\}$$

$$\boldsymbol{f'(x) = -(14x-27)(2x-1)^3(3-x)^2}$$

(4) $f(x) = \dfrac{(1+x)^3(1-2x)}{(1-x)(1+2x)^3}$

これを商の微分で処理するのは大変．

$$\log|f(x)| = 3\log|1+x| + \log|1-2x| - \log|1-x| - 3\log|1+2x|$$
$$\frac{f'(x)}{f(x)} = 3\cdot\frac{1}{1+x} + \frac{-2}{1-2x} - \frac{-1}{1-x} - 3\cdot\frac{2}{1+2x}$$
$$= \frac{-2(4x^2-3x+2)}{(1+x)(1+2x)(1-x)(1-2x)}$$
$$\therefore f'(x) = f(x)\cdot\frac{-2(4x^2-3x+2)}{(1+x)(1+2x)(1-x)(1-2x)}$$
$$= \frac{(1+x)^3(1-2x)}{(1-x)(1+2x)^3}\cdot\frac{-2(4x^2-3x+2)}{(1+x)(1+2x)(1-x)(1-2x)}$$

$$\boldsymbol{f'(x) = -\frac{2(1+x)^2(4x^2-3x+2)}{(1-x)^2(1+2x)^4}}$$

(5) $f(x) = \dfrac{(x+1)^2}{(x+2)^3(x+3)^4}$

全問と同様．

$$\log|f(x)| = 2\log|x+1| - 3\log|x+2| - 4\log|x+3|$$
$$\frac{f'(x)}{f(x)} = \frac{2}{x+1} - \frac{3}{x+2} - \frac{4}{x+3} = \frac{-5x^2-14x-5}{(x+1)(x+2)(x+3)}$$
$$f'(x) = \frac{(x+1)^2}{(x+2)^3(x+3)^4}\cdot\frac{-5x^2-14x-5}{(x+1)(x+2)(x+3)}$$

$$\boldsymbol{f'(x) = -\frac{(x+1)(5x^2+14x+5)}{(x+2)^4(x+3)^5}}$$

(6) $f(x) = x^{\sin x} \quad (x > 0)$

右辺はつねに正なので, 絶対値をとっても変わらない.

$$\log|f(x)| = \sin x \cdot \log x$$

$$\frac{f'(x)}{f(x)} = \cos x \cdot \log x + \sin x \cdot \frac{1}{x}$$

$$\therefore f'(x) = x^{\sin x}\left(\cos x \cdot \log x + \frac{\sin x}{x}\right)$$

$$\boldsymbol{f'(x) = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right)}$$

(7) $f(x) = \sqrt[3]{(x-2)(x^2-2)}$

n 乗根も対数で処理可能.

$$\log|f(x)| = \frac{1}{3}\left\{\log(x-2) + \log(x^2-2)\right\}$$

$$\frac{f'(x)}{f(x)} = \frac{1}{3}\left(\frac{1}{x-2} + \frac{2x}{x^2-2}\right)$$

$$= \frac{1}{3} \cdot \frac{3x^2-4x-2}{(x-2)(x^2-2)}$$

$$f'(x) = \sqrt[3]{(x-2)(x^2-2)} \cdot \frac{1}{3} \cdot \frac{3x^2-4x-2}{(x-2)(x^2-2)}$$

$$= \frac{1}{3} \cdot \frac{3x^2-4x-2}{\{(x-2)(x^2-2)\}^{\frac{2}{3}}}$$

$$\boldsymbol{f'(x) = \frac{3x^2-4x-2}{3\sqrt[3]{(x-2)^2(x^2-2)^2}}}$$

(8) $f(x) = x^{e^x} \quad (x > 0)$

右辺はつねに正なので, 絶対値をとっても変わらない.

$$\log|f(x)| = e^x \log x$$

$$\frac{f'(x)}{f(x)} = e^x \log x + e^x \cdot \frac{1}{x} = e^x\left(\log x + \frac{1}{x}\right)$$

$$\boldsymbol{f'(x) = x^{e^x} e^x\left(\log x + \frac{1}{x}\right)}$$